MATH 2055 Tutorial 1 (Sep 14)

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1. Write down the negations of the following statements.

   (a) $\forall \epsilon > 0$, $\exists N$ such that $\forall n > N$, $|x_n - x| < \epsilon$

   (b) $\exists N$, such that $\forall n > N$, $\forall \epsilon > 0$, $|x_n - x| < \epsilon$

   (c) $\exists M$, such that $\forall \epsilon > 0$, $\exists n$ where $S_n > M - \epsilon$, and $\forall m$, $S_m \leq M$

2. Prove the following statements.

   (a) Let $b < 0$. If x is a number such that $|x - b| < \frac{|b|}{2}$, then $x < \frac{b}{2}$.

   (b) If $a, b \in \mathbb{R}$, then $||a| - |b|| \leq |a - b|$

   (c) If $x, y, z \in \mathbb{R}$ and $x \leq z$, then $x \leq y \leq z$ if and only if $|x - y| + |y - z| = |x - z|$.